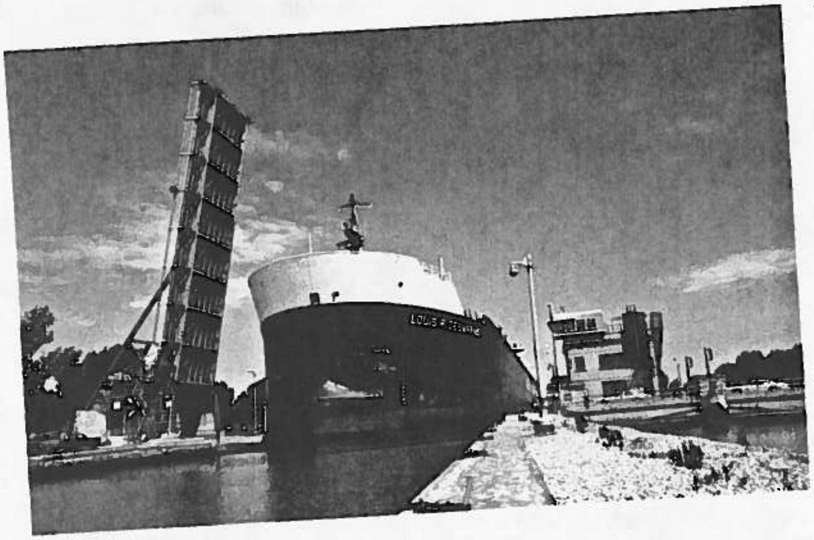
3.1

# Investigate Properties of Triangles



The distribution of an object's mass around its balance point is such that no net force acts to tilt the object. In fact, the object acts like its entire mass is concentrated at a point directly above the balance point. Calculating a balance point accurately can be crucial in the design of aircraft, ships, lift bridges, cranes, and other machinery. Analytic geometry can help find balance points and other properties of geometric shapes.

## Investigate

**How can you find the balance point of a flat triangular object?**

**Tools**
- ruler
- cardboard
- scissors
- compasses

**Method 1: Use Pencil and Paper**

1. Draw a large triangle on a sheet of paper. Then, draw an identical triangle on cardboard and cut out this triangle.
2. Move the cardboard triangle around on the tip of a pencil until you find the point where the triangle balances on the pencil. Mark this balance point. Then, mark the corresponding point on the triangle on the sheet of paper.
3. Fold the paper triangle along one of its medians. Where is the balance point relative to this fold?
4. Explain how you know that the median bisects the area of the triangle. Will the cardboard triangle balance on the edge of a ruler if the median is aligned with the edge? Explain your reasoning. Try balancing the cardboard triangle on the edge of a ruler.
5. Fold the paper triangle in half along a different median. What do you notice about the balance point?
6. Fold the paper triangle along the third median. What do you notice about the creases from the three folds? Check whether your classmates get the same result.

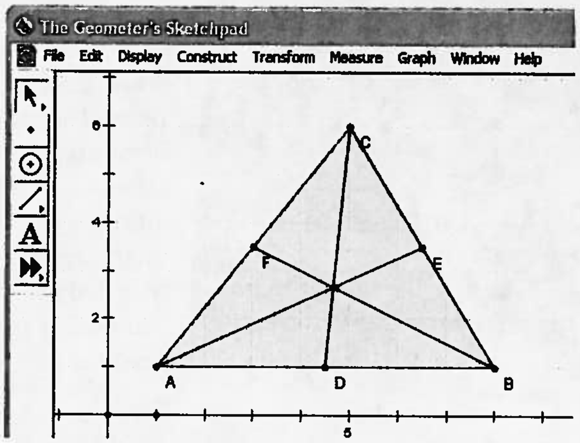
**7. Reflect**

**a)** Are the medians of your triangle **concurrent**? Do you think that all triangles have this property? Explain your reasoning.

**b)** How is the balance point of your triangle related to the **centroid** of the triangle? Do you think that this relationship applies for all triangles? Explain.

**Method 2: Use *The Geometer's Sketchpad®***

**1.** Construct any triangle. Then, construct the three medians of the triangle. Are the medians **concurrent**?




**2.** Observe the intersection of the medians while you drag each of the vertices of the triangle to change the shape of the triangle.

**3. Reflect** Does every triangle have a **centroid**, a single point where all three medians intersect? Explain your reasoning.

**4.** Construct and measure the areas of the two triangles that have median AE as their base. Compare these areas. Then, construct the altitude to AE for both triangles. Compare the lengths of these altitudes.

**5.** Compare the areas and altitudes of the two triangles that have median BF as their base. Then, compare the areas and altitudes of the two triangles that have median CD as their base.

**6.** Drag the vertices of the original triangle around the screen. Do the relationships among the areas and altitudes of the smaller triangles change?

**7. Reflect**

**a)** What property does the median of a triangle have? Explain how you know that all medians have this property.

**b)** Will a flat triangular object balance if it is placed with one of its medians along the edge of a ruler? Explain.

**c)** Where is the balance point of a flat triangular object? Explain your reasoning.

**concurrent**

- meeting at a single point

**centroid**

- the point where the three medians of a triangle intersect

**Tools**

- computer with *The Geometer's Sketchpad®*

**Technology Tip**

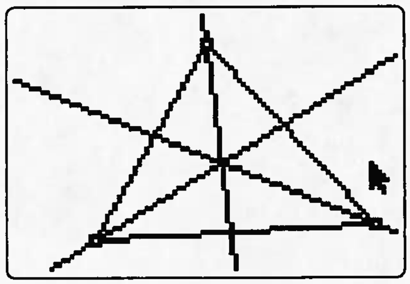
When there are overlapping triangles, you can change which interior is selected by clicking on it again.

**Tools**

- TI-83 Plus or TI-84 Plus graphing calculator

### Method 3: Use a Graphing Calculator

1. Start the Cabri® Jr. application. Choose **Triangle** from the **F2** menu, and draw a large triangle. Choose **Midpoint** from the **F3** menu, and construct the midpoint of each side of the triangle. Choose **Segment** from the **F2** menu, and draw the medians of the triangle by joining each vertex to the midpoint of the opposite side. Are the medians **concurrent** ?



2. Move the cursor to one of the vertices of the triangle. Press ALPHA. Observe the intersection of the medians while you drag the vertex of the triangle to change the shape of the triangle. Try dragging the other vertices as well.

3. **Reflect** Does every triangle have a **centroid** , a single point where all three medians intersect? Explain your reasoning.

4. Construct a new triangle on top of the original triangle by selecting the endpoints of a median and one of the other vertices of the original triangle. Measure the area of the new triangle. Then, construct the other triangle that has the median as its base, and measure the area of this triangle. Compare the areas of the two smaller triangles. How are the altitudes to the common base of the two triangles related?

5. Compare the areas and altitudes of the two triangles that have the second median as their base. Then, compare the areas and altitudes of the two triangles that have the third median as their base.

6. Drag the vertices of the original triangle around the screen. Do the relationships among the areas and altitudes of the smaller triangles change?

7. **Reflect**
   a) What property does the median of a triangle have? Explain how you know that all medians have this property.
   b) Will a flat triangular object balance if it is placed with one of its medians along the edge of a ruler? Explain.
   c) Where is the balance point of a flat triangular object? Explain your reasoning.

**Technology Tip**

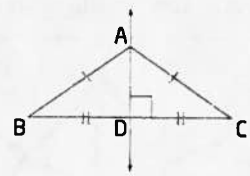
Measure the area of each triangle before constructing the next triangle on top of it. Once a triangle is covered, you cannot select it for measurement.

## Example Median of an Isosceles Triangle

What are the properties of the median from the vertex between the equal sides of an isosceles triangle?

### Solution

Draw any isosceles triangle. Label the equal sides AB and AC. Let D be the midpoint of BC.




The right bisector of a line segment includes all points that are equidistant from the endpoints of the line segment. Since AB = AC, vertex A lies on the right bisector of BC. The midpoint, D, also lies on the right bisector of BC. So, AD is perpendicular to BC.

Since the median from vertex A is perpendicular to BC, this median is also the altitude from vertex A.

$\triangle$ABC and $\triangle$ACD have side AD in common. The other corresponding sides of these two triangles are equal since AB = AC and D is the midpoint of BC. Therefore, $\triangle$ABD and $\triangle$ACD are congruent, and $\angle$BAD = $\angle$CAD. So, AD bisects $\angle$BAC.

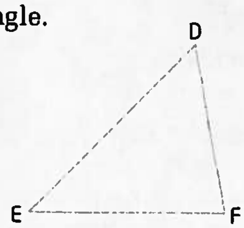
Therefore, the median from the vertex between the equal sides of an isosceles triangle coincides with the altitude to the vertex and bisects the angle at the vertex.

### Key Concepts

- The medians of a triangle meet at a single point, the centroid.
- Each median bisects the area of the triangle.
- The median from the vertex between the equal sides of an isosceles triangle coincides with the altitude to the vertex and bisects the angle at the vertex.
- You can use both pencil-and-paper techniques and geometry software to investigate the properties of geometric shapes.

## Communicate Your Understanding

**C1** Describe two methods for drawing a median of this triangle.




**C2** Explain how you know that this diagram has three pairs of equal angles.